AP® CALCULUS BC
2007 SCORING GUIDELINES (Form B)

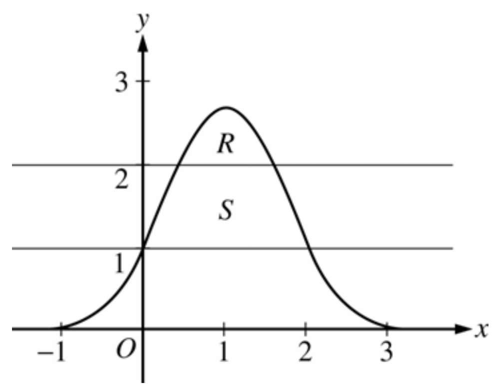
Question 1

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$
Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) $e^{2x-x^2} = 1$ when $x = 0,\ 2$

$$\text{Area of } S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx - \text{Area of } R$$
$$= 2.06016 - \text{Area of } R = 1.546$$

OR

$$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P)\cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$$
$$= 0.219064 + 1.107886 + 0.219064 = 1.546$$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \pi \int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2-1)^2\right) dx$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{constant and limits} \end{cases}$

1A1

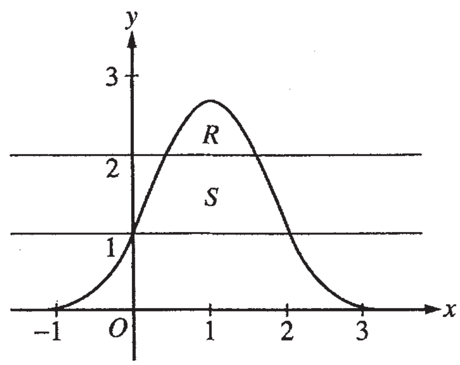
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

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$e^{2x-x^2} = 2 \Rightarrow x = 0.446$ and $x = 1.554$

let $a = 0.446$ and $b = 1.554$

$$\text{Area} = \int_a^b e^{2x-x^2} - 2 \, dx$$

$$= \int_{0.446}^{1.554} e^{2x-x^2} - 2 \, dx$$

$$= 0.514 \text{ unit}^2$$

Continue problem 1 on page 5.

1A$_2$

Work for problem 1(b)

$$\text{Area of } S = \int_0^2 e^{2x-x^2} - 1 \, dx - \text{Area of } R$$

$$= 2.060 - 0.514$$

$$= 1.546 \text{ unit}^2$$

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Work for problem 1(c)

$$V = \pi \int_a^b \left(e^{2x-x^2} - 1\right)^2 - (2-1)^2 \, dx$$

$$\Rightarrow \quad V = \pi \int_{0.446}^{1.554} \left(e^{2x-x^2} - 1\right)^2 - 1 \, dx$$

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1B1

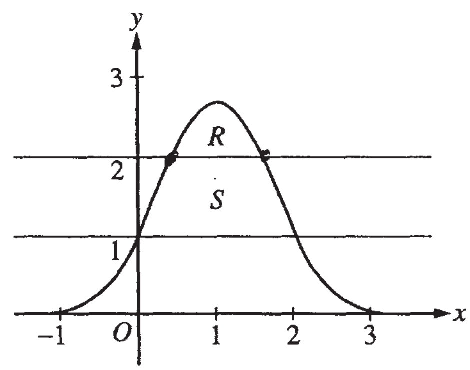
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



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Work for problem 1(a)

(a) $y = 2 = e^{2x - x^2}$

$\ln 2 = 2x - x^2 \Rightarrow x^2 - 2x + \ln 2 = 0$

$x = 1 \pm \sqrt{1 - \ln 2}$

$x = 1.554, 0.446$

$R = \int_{0.446}^{1.554} e^{2x - x^2}\,dx - 2 \times [1.554 - 0.446]$

$= 2.730 - 2.216$

$= 0.514$

$\therefore R = 0.514$

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Continue problem 1 on page 5.

1B₂

Work for problem 1(b)

$y = e^{2x-x^2} = 1$

$\ln 1 = 2x - x^2$

$0 = x(2-x)$

$x = 2, 0$

$S = \int_0^2 e^{2x-x^2}\,dx - R - 2\times 1$

$= 4.060 - 0.514 - 2$

$= 1.546$

$\therefore S = 1.546$

Work for problem 1(c)

$V = \int_0^2 \pi \left(e^{2x-x^2} - 1\right)^2 dx$

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CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

1C

A graphing calculator is required for some problems or parts of problems.

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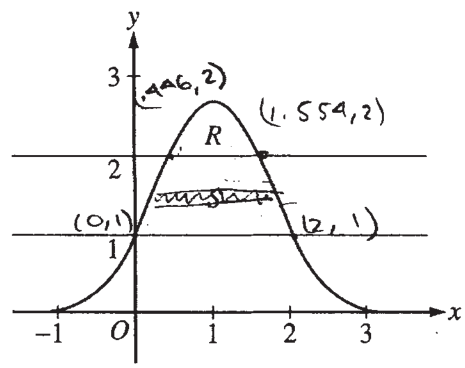

Work for problem 1(a)

a) Area R = $\int_{.446}^{1.554} \left(e^{2x - x^2} - 2\right) dx = 0.514$

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Continue problem 1 on page 5.

 1C2

Work for problem 1(b)

$$\text{Area } S = \int_{.446}^{1.554} (2.718 \cdot e^{2x-x^2}) - (e^{2x-x^2} - 1)\, dx =$$

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Work for problem 1(c)

$$2\pi \int_{.446}^{1.554} (e^{2x-x^2} - 1)^2 \, dx$$

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Question 1

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a), 3 points in part (b), and no points in part (c). Correct work is presented in parts (a) and (b). Although the student attempts a correct solution by rotating the region $R + S$ about $y = 1,$ the response does not subtract the volume obtained when region S is rotated about $y = 1.$ The integrand and the limits are incorrect, so the student did not earn any points in part (c).

Sample: 1C
Score: 3

The student earned 3 points: 3 points in part (a), no points in part (b), and no points in part (c). The student presents correct work in part (a). Incorrect limits and an incorrect integrand are shown in part (b), so no points were earned. In part (c) the student has an incorrect integrand and so did not earn the first 2 points. The correct limits are shown, but the student did not earn the limits and constant point because of the extra factor of 2 multiplied by the integral.

Question 2

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2 + 1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

(a) Speed $= \sqrt{x'(4)^2 + y'(4)^2} = 2.912$	1 : speed at $t = 4$
(b) Distance $= \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 6.423$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) $x(4) = x(0) + \int_0^4 x'(t)\,dt$ $= -3 + 2.10794 = -0.892$	$3 : \begin{cases} 2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = -3 \end{cases} \\ 1 : \text{answer} \end{cases}$
(d) The slope is 2, so $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$, or $\ln\left(t^2 + 1\right) = 2\arctan\left(\frac{t}{1+t}\right)$. Since $t > 0$, $t = 1.35766$. At this time, the acceleration is $\langle x''(t), y''(t)\rangle\big\vert_{t=1.35766} = \langle 0.135, 0.955\rangle$.	$3 : \begin{cases} 1 : \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2 \\ 1 : t\text{-value} \\ 1 : \text{values for } x'' \text{ and } y'' \end{cases}$

BC
2A1

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Work for problem 2(a)

$$\text{speed} = \sqrt{\left[\arctan\left(\frac{t}{1+t}\right)\right]^2 + \left[\ln(t^2+1)\right]^2}$$

$$\text{speed}\Big|_{t=4} = \sqrt{0.45528 + 8.027}$$

$$= 2.912$$

Work for problem 2(b)

$$\int_0^4 \sqrt{\left[\arctan\left(\frac{t}{1+t}\right)\right]^2 + \left[\ln(t^2+1)\right]^2}$$

$$= 6.4233$$

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Continue problem 2 on page

BC
2A₂

Work for problem 2(c)

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right)$$

$$\int dx = \int \arctan\left(\frac{t}{1+t}\right) dt$$

$$x(4) = x(0) + \int_0^4 \arctan\left(\frac{t}{1+t}\right) dt$$

$$= -3 + 2.1079$$

$$= -0.8921$$

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Work for problem 2(d)

$$\frac{dy}{dx} = \frac{\ln(t^2+1)}{\arctan\left(\frac{t}{1+t}\right)}$$

$$2 = \frac{\ln(t^2+1)}{\arctan\left(\frac{t}{1+t}\right)}$$

$$0 = \frac{\ln(t^2+1)}{\arctan\left(\frac{t}{1+t}\right)} - 2$$

$t = 1.3576631$

Let $1.3576631 = c$

$x''(c) = 0.13510$

$y''(c) = 0.955$

$\bar{a}(1.358) = \langle 0.1351, 0.955 \rangle$

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Work for problem 2(a)

The speed $v(t)$ is given by

$v(t) = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$\therefore\ v(4) = \sqrt{[\arctan(0.8)]^2 + [\ln(17)]^2} \approx 2.912$

Work for problem 2(b)

total distance $d(t)$ is given by

$d(t) = \int_0^t v(s)\,ds$

$\therefore\ d(4) = \int_0^4 \sqrt{\left[\arctan\left(\frac{t}{1+t}\right)\right]^2 + [\ln(t^2+1)]^2}\,dt \approx 6.423$

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Continue problem 2 on page

BC
2B2

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Work for problem 2(c)

$x(t) = \int \arctan\left(\frac{t}{1+t}\right) dt$

$x(4) = \int_0^4 \arctan\left(\frac{t}{1+t}\right) dt \approx 2.108$

Work for problem 2(d)

When the slope of the tangent line is 2, $\frac{dy}{dx} \left(= \frac{dy}{dt} \Big/ \frac{dx}{dt}\right) = 2$.

So $\ln(t^2+1) / \arctan\left(\frac{t}{1+t}\right) = 2$, and $t \approx 1.358$.

And the acceleration vector $\vec{a}(t)$ is

$\vec{a}(1.358) = \left.\frac{d^2x}{dt^2}\right|_{t=1.358} \hat{x} + \left.\frac{d^2y}{dt^2}\right|_{t=1.358} \hat{y}$

$= 0.233\,\hat{x} + 0.782\,\hat{y}$.

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BC
2C, 1

Work for problem 2(a)

~~Spee~~ Speed of an object $= \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}$ where $\frac{dx}{dt} = \arctan(\frac{t}{1+t})$

$\frac{dy}{dt} = \ln(t^2+1)$.

Since $t = 4$, $\frac{dx}{dt} = 0.675$

$\frac{dy}{dt} = 2.833$.

$\therefore$ speed $= \sqrt{(0.675)^2 + (2.833)^2}$

$= 2.912$.

$\therefore$ speed $= 2.912$.

Work for problem 2(b)

total distance traveled $\int_a^b \frac{dy}{dx}$ where $\frac{dy}{dt}$ is velocity.

in this case, $\int_0^4 \frac{\ln(t^2+1)}{\tan^{-1}(\frac{t}{1+t})}$ since $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx}$.

$\therefore$ using calculator,

total distance traveled in the time interval $0 \le t \le 4$

$= 9.953$

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Continue problem 2 on page

Bc
2c$_2$

Work for problem 2(c)

$x(t) = \int \arctan\left(\frac{t}{1+t}\right) dt$

$= \int \arctan(t+1) \cdot dt$

using calculator to integrate ~~and~~ substitute $t=4$,

$x(4) = 5.238.$

Work for problem 2(d)

the point the tangent line has slope 2 is when $\frac{dy}{dx} = 2.$

$\therefore \frac{\ln(t^2+1)}{\tan^{-1}\left(\frac{t}{1+t}\right)} = 2.$ $t = -2.588$ or 1.358

but since t cannot be negative.

$t = 1.358$

acceleration vector is $(x''(t), y''(t)).$

$\left(\frac{1}{2x^2+2x+1}, \frac{2x}{x^2+1}\right)$

incase you cannot read.

$x''(t) = \frac{d}{dt}\arctan\left(\frac{t}{1+t}\right)$ / $y''(t) = \frac{d}{dt}\ln(t^2+1)$

acceleration vector

$\therefore \left(\frac{1}{2x^2+2x+1}, \frac{2x}{x^2+1}\right)$

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AP® CALCULUS BC
2007 SCORING COMMENTARY (Form B)

Question 2

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), 1 point in part (c), and 2 points in part (d). Correct work is presented in parts (a) and (b). In part (c) the first point was earned for the correct setup of the integral. The student does not use the initial condition that $x(0) = -3,$ so the last 2 points were not earned. In part (d) the student does not correctly evaluate the components of the acceleration vector, so the last point was not earned.

Sample: 2C
Score: 4

The student earned 4 points: 1 point in part (a), no points in part (b), 1 point in part (c), and 2 points in part (d). Correct work is presented in part (a). In part (b) the student does not use the fact that the distance traveled is found by integrating the speed. In part (c) the first point was earned for a correct integrand. The student does not use the initial condition that $x(0) = -3,$ so the last 2 points were not earned. In part (d) the student was awarded the first 2 points. The first point was earned when the student sets $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2.$ The student does not find the acceleration vector at $t = 1.358,$ and so the third point was not earned. The student could have used the graphing calculator to determine the acceleration vector by the numerical derivative.

AP® CALCULUS BC
2007 SCORING GUIDELINES (Form B)

Question 3

The wind chill is the temperature, in degrees Fahrenheit $(°\text{F})$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°\text{F}$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°\text{F}$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286 When $v = 20$ mph, the wind chill is decreasing at $0.286\ °\text{F}/\text{mph}$.	$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{explanation} \end{cases}$
(b) The average rate of change of W over the interval $5 \le v \le 60$ is $\dfrac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254. $W'(v) = \dfrac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.	$3 : \begin{cases} 1 : \text{average rate of change} \\ 1 : W'(v) = \text{average rate of change} \\ 1 : \text{value of } v \end{cases}$
(c) $\left.\dfrac{dW}{dt}\right\|_{t=3} = \left.\left(\dfrac{dW}{dv} \cdot \dfrac{dv}{dt}\right)\right\|_{t=3} = W'(35) \cdot 5 = -0.892\ °\text{F}/\text{hr}$ OR $W = 55.6 - 22.1(20 + 5t)^{0.16}$ $\left.\dfrac{dW}{dt}\right\|_{t=3} = -0.892\ °\text{F}/\text{hr}$	$3 : \begin{cases} 1 : \dfrac{dv}{dt} = 5 \\ 1 : \text{uses } v(3) = 35, \\ \quad \text{or} \\ \quad \text{uses } v(t) = 20 + 5t \\ 1 : \text{answer} \end{cases}$
Units of $°\text{F}/\text{mph}$ in (a) and $°\text{F}/\text{hr}$ in (c)	1 : units in (a) and (c)

3A₁

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Work for problem 3(a)

$W(v) = 55.6 - 22.1\, V^{0.16}$

$W'(v) = -22.1(0.16)\, V^{0.16-1}$

$= -3.536\, V^{-0.84}$

$W'(20) \approx -3.536(20)^{-0.84}$

$\approx -0.286\ {}^{\circ}F/mph$

It means that the wind chill is decreasing at a rate of $0.286\ {}^{\circ}F/mph$ when $V = 20$ mph.

Work for problem 3(b)

$W'(v) = -3.536\, V^{-0.84}$

avg. rate of change of W

$= \frac{1}{60-5}\int_5^{60} W'(v)\, dV$

$= \frac{1}{55}\int_5^{60} -3.536\, V^{-0.84}\, dV$

$= \frac{1}{55}(-13.95882)$

$\approx -0.254\ {}^{\circ}F/mph$

$W'(v) = -0.254$

$-3.536 V^{-0.84} = -0.254$

$V = 22.989$ mph

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Continue problem 3 on page 9.

3A2

Do not write beyond this border.

Work for problem 3(c)

$\frac{dv}{dt} = 5$

$\int dv = \int 5\, dt$

$v = 5t + c$

$20 = 5(0) + c$

$c = 20$

$\therefore v(t) = 5t + 20$

@ $t = 3$.

$v(3) = 15 + 20 = 35$ mph.

$\frac{dw}{dv} = -3.536 v^{-0.84}$

$\frac{dv}{dt} = 5$

$\frac{dw}{dt} = \frac{dw}{dv} \cdot \frac{dv}{dt}$

$= (-3.536 v^{-0.84})(5)$

$\left.\frac{dw}{dt}\right|_{t=3} = \left[-3.536 (35)^{-0.84}\right](5)$

$\approx \underline{-0.892 \ {}^{\circ}F/h}$

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)

$W(v) = 55.6 - 22.1v^{.16}$

$W'(v) = -22.1(.16)v^{-.84}$

$W'(20) = -.286$ °F/mph. the rate of change of the windchill at $(v = 20)$

Work for problem 3(b)

$$\frac{F(60) - F(5)}{60-5} = \frac{13.0503 - 27.0091}{60-5} = \boxed{-.254}$$

$$W'(v) = -22.1(.16)v^{-.84} = -.254$$

$$\frac{1}{-.84}(v^{-.84}) = (.7177)^{1/-.84}$$

$$\boxed{v = 23.011}$$

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Continue problem 3 on page 9.

 3B2

Work for problem 3(c)

$W(v) = 55.6 - 22.1v^{.16}$

$w'(v) = -22.1(.16)v^{-.84}$

$w'(35) = -.178 \frac{\text{degrees}}{\text{mhp}}$

$\frac{dv}{dt} = 5$

$v @ t = 3 = 35$

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Do not write beyond this border.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

3C1

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Work for problem 3(a)

$W'(v) = .16(-22.1)v^{-.84}$ $v=20$

$W'(20) = .16(-22.1)(20^{-.84})$

$= -.2855$

$\boxed{W'(20) = -.286 \text{ m/hr.}}$

W'(20) is how fast and in which direction the wind chill is moving when the air temperature is felt when the wind is traveling at a velocity of 20 mph

Work for problem 3(b)

average rate change $= \dfrac{W(b) - W(a)}{b-a}$

$= \dfrac{W(60) - W(5)}{b-a}$

$= \dfrac{(55.6 - 22.1(60)^{.16}) - (55.6 - 22.1(5)}{60-5}$

$\dfrac{13.0503 - 27.0091}{55}$

$= -.2537$

avg rate of chang $\boxed{= -.254 \text{ m/hr}}$

Continue problem 3 on

3C2

Work for problem 3(c)

$\frac{dV}{dt} = 5$ m/mr $t = 0$

$V = 20$

$V = 20 + 5t \quad x = 3$

@ $V_{t=3} = 20 + 15$

$= 45$

$$\frac{w(45) - W(20)}{45 - 20}$$

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Question 3

Sample: 3A
Score: 9

The student earned all 9 points. The answer of 22.989 in part (b) is acceptable. In this case the student sets $W'(v)$ equal to the correct average rate of change rounded to three decimal places and correctly solves for v.

Sample: 3B
Score: 6

The student earned 6 points: 1 point in part (a), 3 points in part (b), 2 points in part (c), and no units point. In part (a) $W'(20)$ is correct, but the student does not give a complete explanation. It was necessary for the student to appeal to the fact that the wind chill is decreasing and not merely changing. In part (b) the student calls the function F instead of W but correctly finds the average rate of change. In part (c) the student earned the first 2 points but does not apply the chain rule to come up with the required answer. The student does not use correct units.

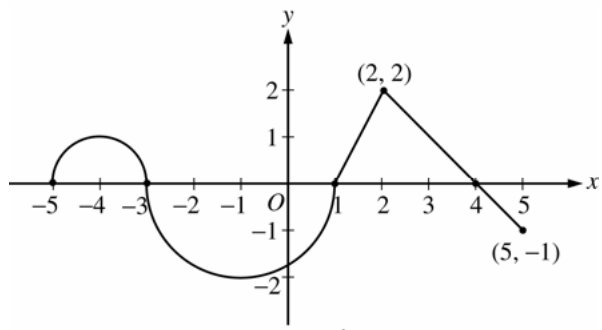
Sample: 3C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and no units point. In part (a) $W'(20)$ is correct, but the student does not explain that the wind chill is decreasing. In part (b) the student earned the first point for the average rate of change. In part (c) the student earned the first point but makes a mistake in calculating the velocity at $t = 3,$ so the second point was not earned. Although the student was eligible for the third point, it was not earned since $\frac{dW}{dt}$ was not found at $t = 3.$ The student does not use correct units.

AP® CALCULUS BC
2007 SCORING GUIDELINES (Form B)

Question 4

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$
f' changes from positive to negative at -3 and 4.
Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1,$ and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1,$ and 2.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{explanation} \end{cases}$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

$3 : \begin{cases} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{cases}$

4 4 4 4 4

NO CALCULATOR ALLOWED

4A1

CALCULUS AB

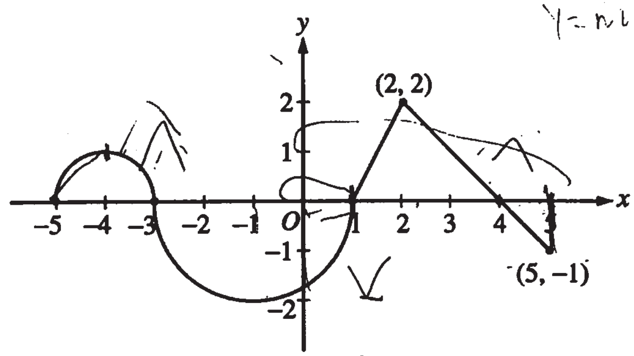
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

$y = mx + b$



Graph of f'

Work for problem 4(a)

a) relative maximum at $x = -3,\ 4$

At $x = -3,\ 4$, the graph of f' change from positive to negative, which hints the graph of f change from increase to decrease. so at $x=-3$, 4, f has relative maximums

Work for problem 4(b)

points of inflection at. $x = -4, -1, 2$

at all these x points, the graph of f' change from increase to decreace or ~~from~~ decrecse to increase, which hints at these points, f change from concave up to concave down or concave down to concave up,

Continue problem 4 on page 11

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4A2

Work for problem 4(c)

when $-5 < x < -4$, $1 < x < 2$,
the graph of f is concave up and also has positive slope.
From the graph of f', when $-5 < x < -4$ and $1 < x < 2$,
the graph of f' is both increasing and above x-axis,
which shows f' and f'' are both positive.
positive f' means the slope of f is positive
and positive f'' means f is concave upward.

Work for problem 4(d)

From the graph of f', the only local
minimum of f is at $x = 1$, $f(1) = 3$

$\int_{-5}^{5} f'(x)\,dx = F(5) - F(-5) = 2\pi - 8\pi + 3 - \frac{1}{2}$
$= \frac{5}{2} - 6\pi < 0$

so $F(5) < F(-5)$

$\int_{1}^{5} f'(x)\,dx = F(5) - F(1) = \frac{3 \times 2}{2} - \frac{1}{2} = \frac{5}{2} > 0$

so $F(5) > F(1)$

thus the absolute minimum value of f(x) over the close interval $-5 \le x \le 5$ is 3.

NO CALCULATOR ALLOWED

4B1

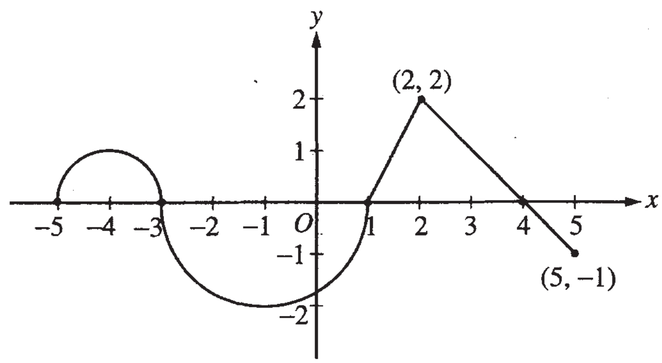
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f'

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Work for problem 4(a)

~~at $x = -4$, and $x = 2$~~

~~at these points, f' changes from increasing to decreasi~~

at $x = -3$, $x = 4$

at these points f' changes from positive to negative

Work for problem 4(b)

at $x = -4$, $x = -1$,

at these points f' changes from increasing to decreasing

Continue problem 4 on page 11

4B$_2$

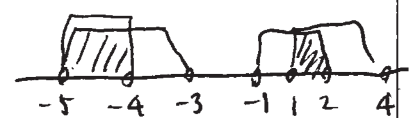
Work for problem 4(c)

f is concave up and has positive slope when $f''(x) > 0$ and $f'(x) > 0$

$f''(x) > 0$ means the slope of f' is positive,

so $f''(x) > 0$ when $(-5, -4)$, $(-1, 2)$,

$f'(x) > 0$ when $(-5, -3)$, $(1, 4)$,



the intervals are $(-5, -4)$, $(1, 2)$

Work for problem 4(d)

$x^2 + y^2 = 1$ $\quad y^2 = 1 - x^2$ $\quad y = \sqrt{1 - x^2}$

f(x) is minimum at the endpoints or at $x = 1$ because f' changes from negative to positive at $x = 1$.

$f(-5) =$

$f(1) = 3$

$f(5) = -\frac{5^2}{2} + 4.5 - \frac{1}{2} = -14$,

$\frac{2+1}{2-5} = -1$ $\quad y - 2 = -(x - 2)$ $\quad f(1) = -\frac{1}{2} + 4 + C = 3$

$C = -\frac{1}{2}$

$y = -x + 4$, $\quad f(x) = \int (-x + 4)\,dx = -\frac{x^2}{2} + 4x + C$

$\frac{-25}{2} + \frac{40}{2} - \frac{1}{2} = -14$

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NO CALCULATOR ALLOWED

4C1

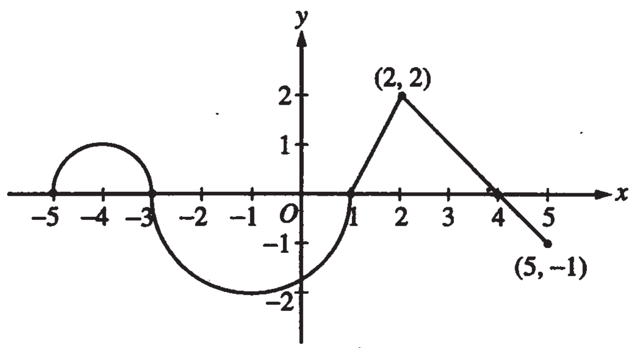
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f'

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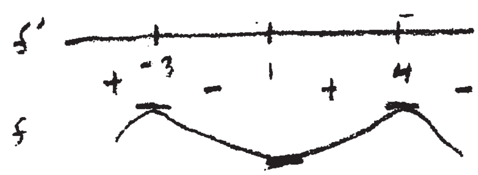
Work for problem 4(a)



$f'(x)$ changes sign at $x = -3, 1, 4$

changes sign from $-$ to $+$ at $x = -3, 4$

$\therefore \quad x = -3, 4$

Work for problem 4(b)

point of inflection occur when $f''(x) = 0$ or is undefined.

$\therefore \; x = -4, -3, -1, 1, 2$

Continue problem 4 on page 11

4C2

Work for problem 4(c)

f': -3, 1, 4

$+ \quad - \quad + \quad -$

f'': -4, -3, -1, 1, 2

$+ \quad - \quad - \quad + \quad + \quad -$

$\therefore (-5,-4), (1,2)$

Work for problem 4(d)

$\frac{1}{2}\pi - \frac{1}{2}4\pi = -\frac{3}{2}\pi$

$f'(x) < 0$ at $-3<x<1$

so

$f(x)$ decrease at $-3<x<1$

$f'(x)<0$ also at $4<x<5$

but

$\left|\int_1^4 f(x)\right| > \left|\int_4^5 f'(x)\right|$

there fore

$f(x)$ have its absolute minimum value at $x=1$

$\boxed{\therefore \ -\frac{3}{2}\pi}$

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Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). Correct work is presented in parts (a) and (c). In part (b) the student only finds two of the three values, so the first point was not earned. The justification point was not earned because it is not true that f' changes from increasing to decreasing at $x = -1.$ In part (d) the student earned the first 2 points since $x = 1$ is identified as a candidate and the endpoints are considered. Since the student never concludes that the absolute minimum is 3, the third point was not earned.

Sample: 4C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d). Correct work is presented in part (a). In part (b) the student gives two additional, incorrect values, so the first point was not earned. No justification is included. In part (c) the first point is earned because of the correct intervals. The student's sign chart alone did not earn the explanation point. It was necessary to explain the reasoning from the sign chart. In part (d) the student earned the first point since $x = 1$ is identified as a candidate. The student does not consider both endpoints and does not give a correct answer, so the last 2 points were not earned.

Question 5

Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Find the values of the constants m, b, and r for which $y = mx + b + e^{rx}$ is a solution to the differential equation.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = -2$. Use Euler's method, starting at $x = 0$ with a step size of $\frac{1}{2}$, to approximate $f(1)$. Show the work that leads to your answer.

(d) Let $y = g(x)$ be another solution to the differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $g(1) \approx 0$. Find the value of k.

(a) $$\frac{d^2y}{dx^2} = 3 + 2\frac{dy}{dx} = 3 + 2(3x + 2y + 1) = 6x + 4y + 5$$

$2 : \begin{cases} 1 : 3 + 2\frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$

(b) If $y = mx + b + e^{rx}$ is a solution, then
$m + re^{rx} = 3x + 2\left(mx + b + e^{rx}\right) + 1.$

If $r \neq 0$: $m = 2b + 1$, $r = 2$, $0 = 3 + 2m$,
so $m = -\frac{3}{2}$, $r = 2$, and $b = -\frac{5}{4}$.

OR

If $r = 0$: $m = 2b + 3$, $r = 0$, $0 = 3 + 2m$,
so $m = -\frac{3}{2}$, $r = 0$, $b = -\frac{9}{4}$.

$3 : \begin{cases} 1 : \frac{dy}{dx} = m + re^{rx} \\ 1 : \text{value for } r \\ 1 : \text{values for } m \text{ and } b \end{cases}$

(c) $$f\left(\frac{1}{2}\right) \approx f(0) + f'(0) \cdot \frac{1}{2} = -2 + (-3) \cdot \frac{1}{2} = -\frac{7}{2}$$
$$f'\left(\frac{1}{2}\right) \approx 3\left(\frac{1}{2}\right) + 2\left(-\frac{7}{2}\right) + 1 = -\frac{9}{2}$$
$$f(1) \approx f\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) \cdot \frac{1}{2} = -\frac{7}{2} + \left(-\frac{9}{2}\right) \cdot \frac{1}{2} = -\frac{23}{4}$$

$2 : \begin{cases} 1 : \text{Euler's method with 2 steps} \\ 1 : \text{Euler's approximation for } f(1) \end{cases}$

(d) $$g'(0) = 3 \cdot 0 + 2 \cdot k + 1 = 2k + 1$$
$$g(1) \approx g(0) + g'(0) \cdot 1 = k + (2k + 1) = 3k + 1 = 0$$
$$k = -\frac{1}{3}$$

$2 : \begin{cases} 1 : g(0) + g'(0) \cdot 1 \\ 1 : \text{value of } k \end{cases}$

NO CALCULATOR ALLOWED

BC5
5A₁

Work for problem 5(a)

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = 3 + 2\cdot\frac{dy}{dx} = 3 + 2(3x+2y+1)$$

$$= 3 + 6x + 4y + 2 = \underline{\underline{6x+4y+5}}$$

Work for problem 5(b)

$y = mx + b + e^{rx}$ $\quad e^{rx} = y - mx - b$

$$\frac{dy}{dx} = m + re^{rx} = m + r(y - mx - b)$$

$$= -rmx + ry + (m - br) = 3x + 2y + 1$$

$-rm = 3$, $r = 2$, $m - br = 1$

$m = \frac{3}{-r} = -\frac{3}{2}$ $\quad br = m - 1 \Rightarrow b = \frac{m-1}{r} = \left(-\frac{3}{2} - 1\right)\cdot\frac{1}{2} = -\frac{5}{2}\cdot\frac{1}{2} = -\frac{5}{4}$

$\therefore m = -\frac{3}{2}$, $r = 2$, $b = -\frac{5}{4}$

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Continue problem 5 on page 13.

 BC5
5A2

NO CALCULATOR ALLOWED

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Work for problem 5(c)

$f(\frac{1}{2}) = f(0+\frac{1}{2}) \approx f(0) + \frac{1}{2}\cdot f'(0) = -2 + \frac{1}{2}(3\cdot 0 + 2\cdot(-2)+1)$

$= -2 + \frac{1}{2}(-4+1) = -2 - \frac{3}{2} = -\frac{7}{2}$

$f(1) = f(\frac{1}{2}+\frac{1}{2}) \approx f(\frac{1}{2}) + \frac{1}{2}\cdot f'(\frac{1}{2}) = -\frac{7}{2} + \frac{1}{2}(3\cdot\frac{1}{2} + 2\cdot(-\frac{7}{2})+1)$

$= -\frac{7}{2} + \frac{1}{2}\left(\frac{3}{2} - 7 + 1\right) = -\frac{7}{2} + \frac{1}{2}\cdot(-\frac{9}{2}) = -\frac{7}{2} - \frac{9}{4}$

$= -\frac{23}{4}$ $\quad \therefore f(1) \approx -\frac{23}{4}$

Work for problem 5(d)

$g(1) = g(0+1) \approx g(0) + 1\cdot g'(0) = k + 1\cdot(3\cdot 0 + 2\cdot k + 1)$

$= k + 2k + 1 = 3k+1 = 0 \Rightarrow 3k = -1$

$k = -\frac{1}{3}$ $\quad \therefore k = -\frac{1}{3}$

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Work for problem 5(a)

$$\frac{d^2y}{dx^2} = 3+2x\frac{dy}{dx}$$
$$= 3+2(3x+2y+1)$$
$$= 3+6x+4y+2$$
$$= 6x+4y+5$$

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Work for problem 5(b)

$$\frac{dy}{dx} = m+re^{rx}$$
$$= 3x+2y+1$$

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Continue problem 5 on page 13.

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Work for problem 5(c)

$f(0) = -2$

$f(0.5) = f(0) + f'(0) \times 0.5$

$= -2 + (-3) \times 0.5$

$= -3.5$

$f(1) = f(0.5) + f'(0.5) \times 0.5$

$= 3.5 + (-3) \times 0.5$

$= -5$

$\therefore f(1) \approx -5$

Work for problem 5(d)

$g(0) = k$

$g(0.5) = g(0) + g'(0) \times 1$

$= k + (2k+1)$

$= 0$

$3k + 1 = 0$

$\therefore k = -\frac{1}{3}$

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GO ON TO THE NEXT PAGE.

BC
$5C_1$

NO CALCULATOR ALLOWED

Work for problem 5(a)

$$\frac{dy}{dx} = 3x + 2y + 1$$

$$\frac{d^2y}{dx^2} = 3 + 2\frac{dy}{dx} = 3 + 2\cdot(3x+2y+1)$$

$$= 3 + 6x + 4y + 2$$

$$= \underline{6x+4y+5}$$

Work for problem 5(b)

$$dy = (3x+2y+1)\,dx \Rightarrow \int dy = \int (3x+2y+1)\,dx$$

$$y = \frac{3}{2}x^2 + 2xy + x$$

$$y - 2xy = y(1-2x) = \frac{3}{2}x^2 + x$$

$$y = \frac{3x^2+2x}{(1-2x)2} = \frac{3x^2+2x}{2-4x} = mx + b + e^{rx}$$

$$3x^2 + 2x = 2mx + 2b + 2e^{rx}$$
$$-4mx^2 - 4bx - 4xe^{rx}$$

$$2b + 2e^{rx} = 0$$

$$m = -\frac{4}{3}$$

$$2 = -\frac{8}{3} - 4b - 4e^{rx}$$

$$-b = e^{rx}$$

$$-4b - 4e^{rx} = \frac{14}{3}$$

$$2x = 2mx - 4bx - 4xe^{rx}$$

Continue problem 5 on page 13.

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Work for problem 5(c)

$Y_0 = -2 \quad x_0 = 0$

$x = 1 \quad y = ?$

$m = -\frac{4}{3}$ from 5(b)

$x_0 + \frac{1}{2} = \frac{1}{2} \quad Y_0 = -2$

$Y_1 = Y_0 + \frac{1}{2}(Y_0') \qquad Y_1' = -\frac{4}{3}$

$= -2 + \frac{1}{2}\left(-\frac{4}{3}\right)$

$= -\frac{8}{3}$

$-\frac{10}{3}$

$x_1 + \frac{1}{2} = 1 = x$

$Y_2 = Y_1 + \frac{1}{2}\left(-\frac{4}{3}\right) = -\frac{8}{3} - \frac{2}{3} = -\frac{10}{3}$

Work for problem 5(d)

$x_0 = 0 \quad Y_0 = k$

$x_0 + \frac{1}{2} = x_1 = \frac{1}{2}$

$x_1 + \frac{1}{2} = x_2 = 1 \qquad Y_1 = Y_0 + \frac{1}{2}(Y_0') = k - \frac{2}{3}$

$Y_2 = \left(k - \frac{2}{3}\right) + \frac{1}{2} \cdot -\frac{4}{3}$

$= k - \frac{4}{3} = 0$

$k = \frac{4}{3}$

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). The student presents correct work in parts (a) and (d). In part (b) the student only finds the correct $\frac{dy}{dx}$, and so the first point was earned. In part (c) the student earned the first point by the use of Euler's method with two steps to approximate $f(1)$. The student makes an error in calculating $f'\left(\frac{1}{2}\right)$, so the second point was not earned.

Sample: 5C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), 1 point in part (c), and no points in part (d). The student presents correct work in part (a). In part (b) the student does not find $\frac{dy}{dx}$. In part (c) the student earned the first point by the use of Euler's method with two steps to approximate $f(1)$. The student makes an error in calculating $f'\left(\frac{1}{2}\right)$, so the second point was not earned. In part (d) the student uses $\frac{1}{2}$ instead of 1 for Δx and makes computational errors, so no points were awarded.

Question 6

Let f be the function given by $f(x) = 6e^{-x/3}$ for all x.

(a) Find the first four nonzero terms and the general term for the Taylor series for f about $x = 0$.

(b) Let g be the function given by $g(x) = \int_0^x f(t)\,dt$. Find the first four nonzero terms and the general term for the Taylor series for g about $x = 0$.

(c) The function h satisfies $h(x) = k\,f'(ax)$ for all x, where a and k are constants. The Taylor series for h about $x = 0$ is given by

$$h(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots.$$

Find the values of a and k.

(a)
$$f(x) = 6\left[1 - \frac{x}{3} + \frac{x^2}{2!3^2} - \frac{x^3}{3!3^3} + \cdots + \frac{(-1)^n x^n}{n!3^n} + \cdots\right]$$
$$= 6 - 2x + \frac{x^2}{3} - \frac{x^3}{27} + \cdots + \frac{6(-1)^n x^n}{n!3^n} + \cdots$$

$3 : \begin{cases} 1 : \text{two of } 6, -2x, \dfrac{x^2}{3}, -\dfrac{x^3}{27} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \end{cases}$

$\langle -1 \rangle$ missing factor of 6

(b) $g(0) = 0$ and $g'(x) = f(x)$, so
$$g(x) = 6\left[x - \frac{x^2}{6} + \frac{x^3}{3!3^2} - \frac{x^4}{4!3^3} + \cdots + \frac{(-1)^n x^{n+1}}{(n+1)!3^n} + \cdots\right]$$
$$= 6x - x^2 + \frac{x^3}{9} - \frac{x^4}{4(27)} + \cdots + \frac{6(-1)^n x^{n+1}}{(n+1)!3^n} + \cdots$$

$3 : \begin{cases} 1 : \text{two terms} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \end{cases}$

$\langle -1 \rangle$ missing factor of 6

(c) $f'(x) = -2e^{-x/3}$, so $h(x) = -2k\,e^{-ax/3}$

$$h(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots = e^x$$

$$-2k\,e^{-ax/3} = e^x$$

$$\frac{-a}{3} = 1 \text{ and } -2k = 1$$

$$a = -3 \text{ and } k = -\frac{1}{2}$$

OR

$f'(x) = -2 + \frac{2}{3}x + \cdots$, so

$$h(x) = kf'(ax) = -2k + \frac{2}{3}akx + \cdots$$

$$h(x) = 1 + x + \cdots$$

$$-2k = 1 \text{ and } \frac{2}{3}ak = 1$$

$$k = -\frac{1}{2} \text{ and } a = -3$$

$3 : \begin{cases} 1 : \text{computes } k\,f'(ax) \\ 1 : \text{recognizes } h(x) = e^x, \\ \qquad \text{or} \\ \quad \text{equates 2 series for } h(x) \\ 1 : \text{values for } a \text{ and } k \end{cases}$

NO CALCULATOR ALLOWED

6A₁

Work for problem 6(a)

$f(x) = 6e^{-\frac{x}{3}}$

$f'(x) = 6\cdot\left(-\frac{1}{3}\right)e^{-\frac{x}{3}}$

$f''(x) = 6\cdot\left(-\frac{1}{3}\right)^2 e^{-\frac{x}{3}}$

$\vdots$

$f^{(n)}(x) = 6\cdot(-1)^n\cdot\left(\frac{1}{3}\right)^n e^{-\frac{x}{3}}$

$= \boxed{6 - 2x + \frac{1}{3}x^2 - \frac{1}{27}x^3}$

$P(x) = \boxed{6 + 6\left(-\frac{1}{3}\right)x + \frac{6\cdot\left(-\frac{1}{3}\right)^2}{2!}x^2 + \frac{6\cdot\left(-\frac{1}{3}\right)^3}{3!}x^3}$ ← the first four nonzero terms

$G(x) = \boxed{\sum_{n=0}^{\infty} \frac{6\cdot(-1)^n}{n!\cdot 3^n}x^n}$ ← the general ~~term~~ term for the Taylor series for f about $x=0$.

Work for problem 6(b)

$g(x) = \int_0^x f(t)\,dt$

$= \int_0^x \sum_{n=0}^{\infty} \frac{6(-1)^n}{n!\cdot 3^n} t^n\,dt$

$= \Big|_0^x\ 6t + 6\left(-\frac{1}{3}\right)\frac{t^2}{2} + \frac{6\cdot\left(-\frac{1}{3}\right)^2}{2!}\frac{t^3}{3} + \frac{6\cdot\left(-\frac{1}{3}\right)^3}{3!}\frac{t^4}{4} + \cdots$

$= \boxed{6x - x^2 + \frac{1}{9}x^3 - \frac{1}{4\times 27}x^4}$ ← the first four nonzero terms

$= \boxed{\sum_{n=0}^{\infty} \frac{6(-1)^n}{n!\cdot 3^n}\frac{x^{n+1}}{n+1}}$ ← the general term for the Taylor series for f about $x=0$.

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Continue problem 6 on page 15.

Work for problem 6(c)

~~$h(x) = e^x$~~

We can know that $h(x) = e^x$.

$f'(x) = 6 \cdot (-\frac{1}{3}) e^{-\frac{x}{3}}$

$\therefore f'(ax) = -2 e^{-\frac{a}{3}x}$

$\therefore h(x) = k f'(ax)$

$\Rightarrow e^x = k \cdot (-2) e^{-\frac{a}{3}x}$

$\Rightarrow e^{(1+\frac{a}{3})x} = -2k$

Since a, k should be independent from x,

$1 + \frac{a}{3} = 0, \quad 1 = -2k$

$\therefore \boxed{a = -3, \ k = -\frac{1}{2}}$

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AC6
6B₁

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(x)}{n!}(x-a)^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$6e^{-\frac{x}{3}} = 6\sum_{n=0}^{\infty} \frac{\left(-\frac{x}{3}\right)^n}{n!} = \boxed{6\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n n!}}$$

27×6

ii) First four

$$\boxed{6,\ -2x,\ \frac{x^2}{3},\ -\frac{x^3}{27}}$$

Work for problem 6(b)

$$g(x) = \int_0^x f(t)\,dt$$

$u = -\frac{t}{3}$
$du = -\frac{1}{3}dt$

$$g(x) = 6\int_0^x e^{-\frac{t}{3}}\,dt$$

$$= -18\int e^u\,du = \left[-18e^{\left(-\frac{t}{3}\right)}\right]_0^x = \boxed{-18e^{\left(-\frac{x}{3}\right)} + 18}$$

$$-18e^{\left(-\frac{x}{3}\right)} = -18\sum_{n=0}^{\infty}\frac{\left(-\frac{x}{3}\right)^n}{n!} = \boxed{-18\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{3^n n!} + 18}$$

ii) First 4 Terms

$$0,\ \frac{+\cancel{18}^{6}\cdot -1\,x}{\cancel{3}} = \boxed{6x+18}\quad \frac{-\cancel{18}\cdot x^2}{\cancel{9\cdot 2}} = \boxed{-x^2+18},\ \frac{+\cancel{18}^{3}\cdot -1\,x^3}{9\ \cancel{27\cdot 6}} = \boxed{\frac{x^3}{9}+18},\ \frac{-\cancel{18}^{9}\,x^4}{9\ \cancel{81\cdot 24}_{2}} = \boxed{-\frac{x^4}{18}+18}$$

Continue problem 6 on page 15.

BC6
6B2

NO CALCULATOR ALLOWED

Work for problem 6(c)

i) $h(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$

ii) $f'(x) = 6 \cdot -\frac{1}{3} e^{\frac{-x}{3}} = -2e^{\frac{-x}{3}}$

$f'(ax) = -2e^{\frac{-ax}{3}}$

$e^x = -2k\, e^{\frac{-ax}{3}}$

$\boxed{\therefore k = -\frac{1}{2}, \quad a = -3}$

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BC6
6C1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$f(x) = 6e^{-x/3} = 6e^{-\frac{x}{3}}$

$T(0) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots \frac{f^n(0)}{n!}$

$= 6 - 2 + \frac{2}{3} - \frac{2}{9} + \dots + (-1)^n \frac{2}{3^{n-1}} + \dots$

a) $f(0) = 6e^0 = 6$

$f'(x) = 6e^{-\frac{x}{3}} \cdot -\frac{1}{3}$

$f'(0) = -2$

$T(0) = 6 - \frac{2x}{1!} + \frac{2x^2}{3\,2!} - \frac{2x^3}{9\,3!} + \dots + \frac{(-1)^n 2x^n}{3^{n-1} n!} + \dots$

$f''(x) = -2e^{-\frac{x}{3}} \cdot -\frac{1}{3}$

$f''(0) = \frac{2}{3}$

$f'''(x) = \frac{2}{3} e^{-\frac{x}{3}} \cdot -\frac{1}{3}$

$f'''(0) = -\frac{2}{9}$

Work for problem 6(b)

$g(x) = \int_0^x f(t)\,dt$

b)

$g(x) = \int_0^x 6 + 2x + \frac{x^2}{3} - \frac{x^3}{27} + \dots + \frac{(-1)^n 2x^n}{3^{n-1} n!} + \dots dx$

$g(x) = 6x + x^2 + \frac{x^3}{9} - \frac{x^4}{108} + \dots + \frac{(-1)^n 2n x^{n-1}}{3^{n-1} n!} + \dots dx$

$\frac{2}{54} \times x = 4$

$2x = 216$

$x = 108$

$4\overline{)108}$ 27

$2\overline{)54}$ 27

$\frac{1}{27} \times x = 4$

$x =$

Continue problem 6 on page 15.

Work for problem 6(c)

$h(x) = kf'(ax)$

$h(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \frac{x^n}{n!} + \cdots$

$f'(x) = -\frac{1}{3}6e^{-\frac{x}{3}}$

$f''(x) = \frac{1}{9}6e^{-\frac{x}{3}}$

$= \frac{2}{3}e^{-\frac{x}{3}}$

$1 = kf'(ax)$

$1 = k(-\frac{1}{3}6e^{-\frac{x}{3}})$

$-\ 1 = -k2e^{-\frac{x}{3}}$

$-1 = k2e^{-\frac{x}{3}}$

$\frac{-1}{2e^{-x/3}} = k$

$k = -\frac{1}{2}e^{x/3}$

$h'(x) = kf''(ax)\cdot a$

$x = k\frac{2}{3}e^{-ax/3} \cdot a$

$x = (-\frac{e^{x/3}}{\cancel{x}})\frac{x}{3}\frac{a}{e^{\frac{ax}{3}}}$

$-3x = \frac{\cancel{\ } e^{x/3}\ a}{e^{\frac{ax}{3}}}$

$-3x = ae^{x/3 - \frac{ax}{3}}$

$-3x = ae^{\frac{x(1-a)}{3}}$

$-\ln 3x = a\frac{x(1-a)}{3}$

$-3\ln 3x = ax(1-a)$

$= ax - a^2x$

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Question 6

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 3 points in part (a), no points in part (b), and 3 points in part (c). Correct work is presented in parts (a) and (c). In part (b) the student adds 18 to each of the first four nonzero terms and thus did not earn the first 2 points. The student does not correctly integrate the general term from part (a), and so the third point was not earned.

Sample: 6C
Score: 4

The student earned 4 points: 3 points in part (a), 1 point in part (b), and no points in part (c). Correct work is presented in part (a). In part (b) the first, third, and fourth terms are correct, but the student makes an error on the sign of the second term. The student earned 1 of the first 2 points. The general term from part (a) is not correctly integrated, and so the third point was not earned. In part (c) the student does not compute $k\,f'(ax)$ correctly and does not recognize $h(x)$ as the series for e^x.